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Calculation of the Invariant Density of the Chaotic Mapping in a DC-DC Converter and Its Applications

POWER ELECTRONICS

Abstract: This paper concerns applying probability analysis to chaotic mapping in DC-DC converters. A computation method for the invariant density of a chaotic mapping is proposed by using eigenvector method. Moreover, the power spectral density of the input to the DC-DC converter and the average frequency of switching are deduced. Further, the invariant density is used for an accurate design of the DC-DC converter parameters. Finally, some examples are given to illustrate the effectiveness of the proposed method.

Keywords: Chaos, DC-DC converter, invariant density, power spectral density

1 Introduction

In the past four decades, chaotic phenomena and chaos theory have been attracting increasing research interest, and many chaos control methods have been proposed, such as the well-known OGY control method, proposed by Ott, Grebogi and Yorke in 1990 [1]. The OGY method can convert a chaotic motion to a desired periodic orbit by making only small perturbations in an accessible set of system parameters. In DC-DC converters, there often appear some singular or abnormal phenomena, such as the unexpected collapse of operating mode, strange electromagnetic interference, unstable operation and failure after a long time running, which can result in system failure and external random disturbance, consequently, greatly limit the applications of DC-DC converters. The study results indicate that the phenomena mentioned above are related to chaotic behavior [2, 3]. It is known that the chaotic motion is an unstable, aperiodic behavior in a bounded area, and its long-term behavior shows random-like characteristics. Thus, it is possible to describe and study the chaotic behavior with probability theory.

The invariant density is a basic and important characteristic of chaos. For a DC-DC converter, a one-dimensional mapping can be derived with some reasonable assumptions, which can then be used to analyze the chaotic behavior of the DC-DC converter. There have been several methods proposed to calculate the invariant density of the chaotic mapping of the DC-DC converter. However, they have their own drawbacks. For instance, the method presented in [4] is difficult to be realized by computer due to the immense increase of the calculation complexity as the iteration of the mapping increases slightly. Moreover, this method does not require the mapping to have a finite number of Markov partitions [5]. The method in [6] uses Frobenius-Perron operator equation to calculate the invariant density. But it is well known that very few Frobenius-Perron operator equations of chaotic mappings can be solved analytically, thus this method can only be applied in a few special cases.

In this paper, a Boost converter operating in a chaotic mode is first represented as a one-dimensional mapping, based on which the invariant density of the chaotic mapping is then calculated using the eigenvector method. The comparison of the invariant density of the chaotic mapping with its phase

the interval $[0, \alpha]$, and 2) ergodic and asymptotically stable [8]. Due to the random-like characteristic of chaos, the eigenvector method, which is extracted from probability theory and will be introduced in the section 4, is employed here to calculate the invariant density of a chaotic mapping.

3 Invariant density of a chaotic mapping

Chaos is a kind of unstable behavior in a bounded area. Its long-term behavior shows random-like characteristics. Thus, it is possible to characterize it with probability theory, namely, using the invariant density $\rho(x)$ of the chaotic mapping. The term “invariant” means that the number of orbit points of chaotic mapping is invariant under the iterations of the mapping [9].

For some simple cases, such as the parabola mapping, it is possible to get the analytic solution. But for general cases, calculating $\rho(x)$ needs to employ the Perron-Frobenius equation to get numerical solutions.

The Perron-Frobenius equation is based on “conservation of quantity” [9]. Fig. 3 shows a nonlinear function, where y has two inverse images x_1 and x_2 , namely, $y = f(x_1) = f(x_2)$.

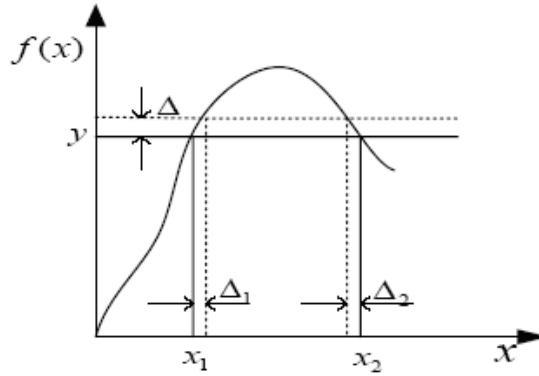


Fig. 3: the mapping of nonlinear function

Denote the small neighborhoods of x_1 , x_2 and y as Δ_1 , Δ_2 and Δ , respectively, and the corresponding probability densities as $\rho(x_1)$, $\rho(x_2)$ and $\rho(y)$. According to the conservation law of quantity [9], one has

$$\rho(y)\Delta = \rho(x_1)\Delta_1 + \rho(x_2)\Delta_2. \quad (2)$$

When Δ_1 , Δ_2 and Δ are small enough, (2) can be recast as,

$$\rho(y) = \frac{\rho(x_1)}{|f'(x_1)|} + \frac{\rho(x_2)}{|f'(x_2)|}, \quad (3)$$

where, $f'(x_1) = \frac{\Delta}{\Delta_1}$, and $f'(x_2) = \frac{\Delta}{\Delta_2}$. If $f(x)$ has multiple inverse images (more than 2), namely, there exist $x_i = f^{-1}(y)$, $i > 2$, (3) can be denoted as

$$\rho(y) = \sum_{\{x_i=f^{-1}(y)\}} \frac{\rho(x_i)}{|f'(x_i)|}. \quad (4)$$

This is the so-called Perron-Frobenius equation, on which the calculation of the invariant density using eigenvector method can be based.

4 Calculating invariant density of chaotic mapping with eigenvector method

For a nonlinear function $f(x)$, $f : I \rightarrow I$, the interval I can be equally divided into M segments. If M is large enough, $\rho(x)$ can be regarded as “invariant” in each small interval. Then, $\rho(x)$ can be expressed as M discrete values $\rho(x_1), \rho(x_2), \dots, \rho(x_M)$ or the vector form $R = [\rho(x_1), \rho(x_2), \dots, \rho(x_M)]$ [10].

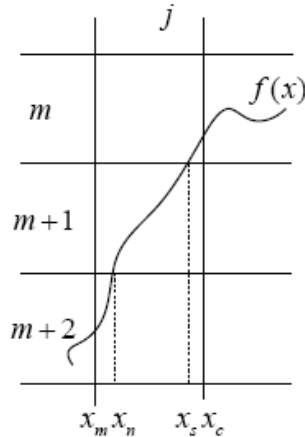


Fig. 4: the partial sketch mapping of chaotic mapping

In Fig.4, $p_{i,j}$ is the transition probability of the j th interval, and the transition probability matrix is denoted by

$$P = \begin{vmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,M} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ p_{M,1} & p_{M,2} & \cdots & p_{M,M} \end{vmatrix}, \quad (5)$$

in which the entries can be derived by

$$\begin{cases} p_{m,j} = (x_n - x_m)/L, \\ p_{m+1,j} = (x_s - x_n)/L, \\ p_{m+2,j} = (x_c - x_s)/L, \\ p_{i,j} = 0 (1 \leq i \leq M, i \neq m, m+1, m+2). \end{cases} \quad (6)$$

Thus, it is easy to see that the calculation of the transition probability matrix P is easy as long as $f(x)$ and M are known. From the definitions of P , R and Perron-Frobenius equation, P and R satisfy the following equality,

$$PR = R. \quad (7)$$

It is concluded from (7) that R is the eigenvector of P with eigenvalue 1. Thus, the calculation of the invariant density is reduced to a calculation of the eigenvector of the transition probability matrix P .

5 Invariant density of the chaotic mapping of the Boost converter

For the above mentioned Boost converter, according to Eqs.(1)–(7), and dividing the interval $[0 \ \alpha]$ into M equal segments, the eigenvector $R = [\rho(x_1), \rho(x_2), \dots, \rho(x_M)]$ of P , namely, the invariant

density of the chaotic mapping, can be calculated. When α samples different values, the simulation results are presented in the following.

When $\alpha = 1.30$, the phase portrait of the mapping is showed in Fig. 5(a), and the corresponding bifurcation diagram and the invariant density are showed in Fig. 5(b) and Fig. 5(c). From these figures, it is obviously to see that they inosculate quite well. It is remarked that the invariant density reflects the operating status of the Boost converter from another angel.

It is seen from Fig. 5(a) that there are no orbit points in the intervals $[0.13, 0.91]$ and $[1.10, 1.15]$, corresponding to the zero invariant density in those intervals. Similarly, for the cases $\alpha = 1.52$, and $\alpha = 2.65$, the simulation results are illustrated in Figs. 5, 6 and 7:

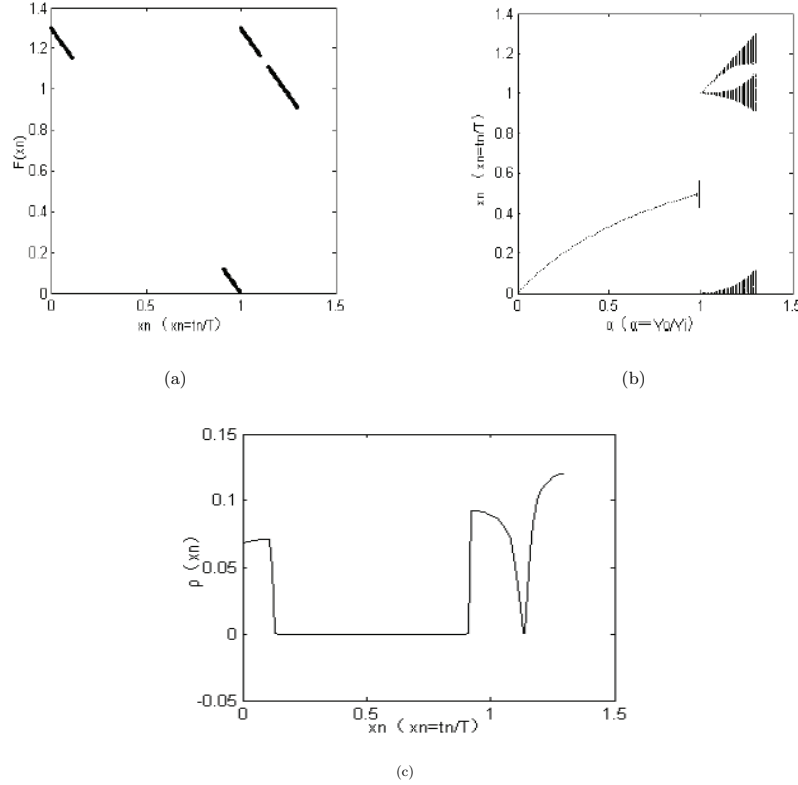


Fig. 5: The chaotic mapping (a), bifurcation diagram (b), and the invariant density (c) at $\alpha = 1, 30$

The simulation results have illustrated the accuracy of the eigenvector method in calculating the invariant density.

6 Applications examples of invariant density

The invariant density can be used to calculate the power spectral density of the input of the DC-DC converter, estimate the average switching frequency and accurately design the parameters of the DC-DC converter. Two examples are given in the following for illustration.

6.1 Power Spectral Density of the input in a DC-DC converter

Consider the above introduced Boost converter. According to Fig.2, the quadratic derivative of inductor current is showed in Fig.8. According to [11], the inductor current can be expressed by

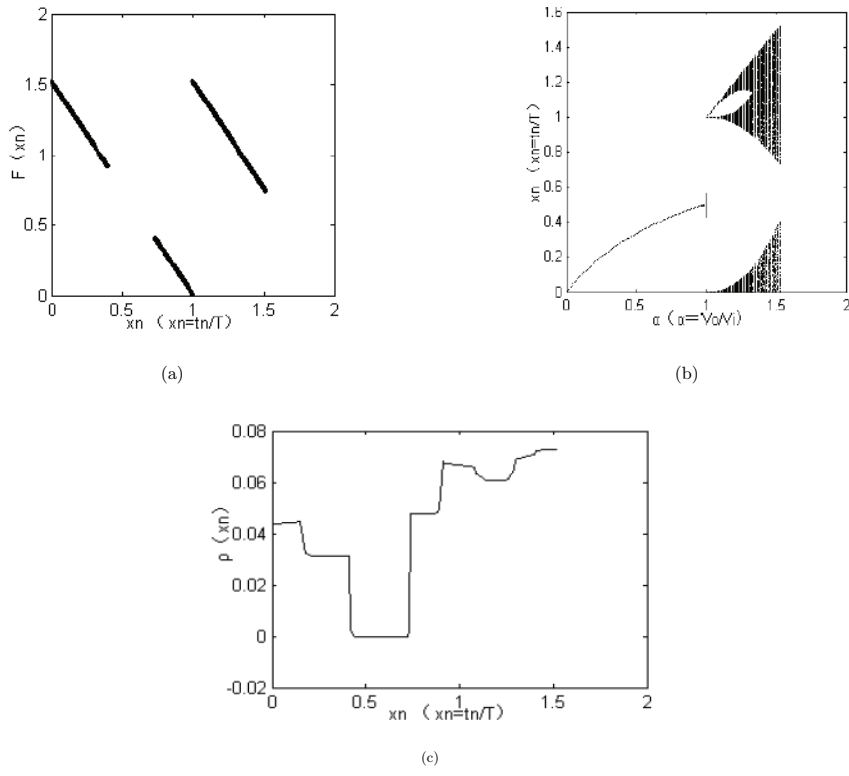


Fig. 6: The chaotic mapping (a), bifurcation diagram (b), and the invariant density (c) at $\alpha = 1, 52$

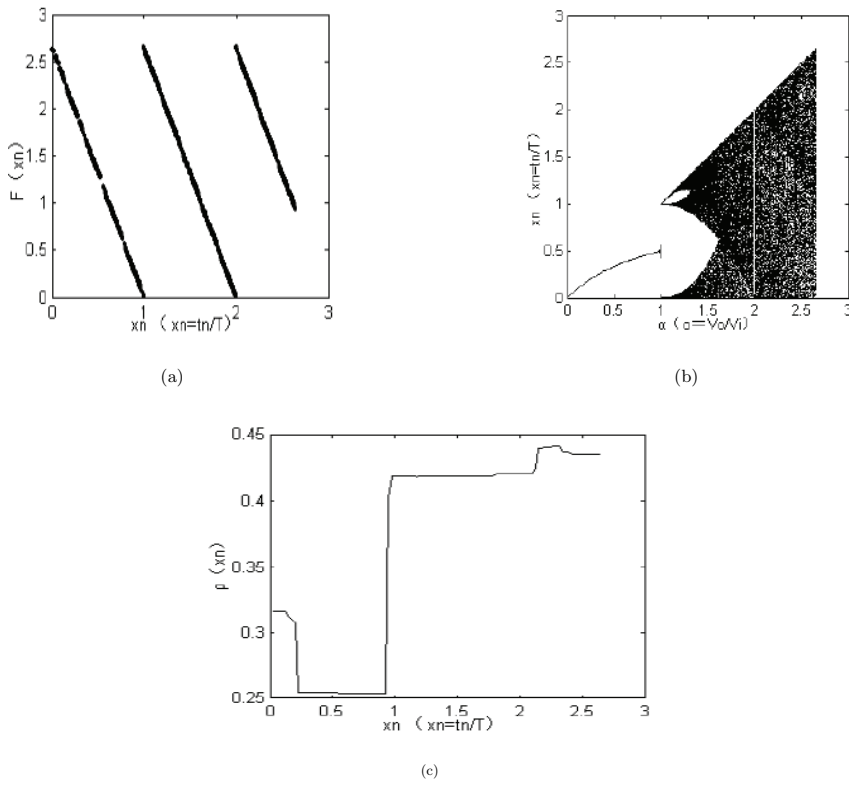


Fig. 7: The chaotic mapping (a), bifurcation diagram (b), and the invariant density (c) at $\alpha = 2, 65$

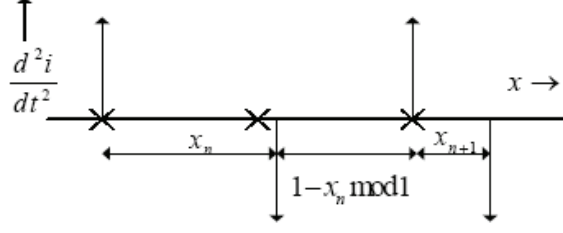


Fig. 8: Quadratic derivative of inductor current

$$\begin{aligned} \frac{d^2 i}{dt^2} = & -\frac{\bar{V}_O}{L} \{ \delta(t) - \delta(t - T_C x_1) + \delta[t - T_C(1 + \lfloor x_1 \rfloor)] - \delta[t - T_C(1 + \lfloor x_1 \rfloor + x_2)] \\ & + \cdots - \delta[t - T_C(N - 1 + \sum_{k=1}^{N-1} \lfloor x_k \rfloor) + x_N] + \delta[t - T_C(N + \sum_{k=1}^N \lfloor x_k \rfloor)] \}, \end{aligned} \quad (8)$$

where $\lfloor x \rfloor$ means the round-off number. Employing the following Fourier transformation,

$$g(t) \Leftrightarrow G(\omega) \Rightarrow \int_{-\infty}^t g(u) du \Leftrightarrow \frac{1}{j\omega} G(\omega),$$

$$g(t) \Leftrightarrow G(\omega) \Rightarrow g(t - \tau) \Leftrightarrow e^{-j\omega\tau} G(\omega),$$

and

$$\delta(t) \Leftrightarrow 1,$$

results in the Fourier transform of the inductor current as

$$\begin{aligned} A(\omega) = & -\frac{\bar{V}_O}{\omega^2 L} \lim_{N \rightarrow \infty} \frac{1}{T_N} [\{1 - \exp(-j\omega T_C x_1)\} + \exp(-j\omega T_C [1 + \lfloor x_1 \rfloor]) \{1 - \exp(j\omega T_C x_2)\} \\ & + \cdots + \exp(-j\omega T_C [N - 1 + \sum_{k=1}^{N-1} \lfloor x_k \rfloor]) (1 - \exp(-j\omega T_C x_N))]. \end{aligned} \quad (9)$$

With the denotation

$$J_n = \begin{cases} 0 & \text{for } n = 1 \\ \sum_{k=1}^{N-1} 1 + \lfloor x_k \rfloor = n - 1 + \sum_{k=1}^{N-1} \lfloor x_k \rfloor & \text{for } n > 1 \end{cases}, \quad (10)$$

and

$$T_n = \sum_{n=1}^N 1 + \lfloor x_n \rfloor, \quad (11)$$

the equation (9) can be rewritten as

$$A(\omega) = -\frac{\bar{V}_O}{\omega^2 L} \lim_{N \rightarrow \infty} \frac{1}{T_N} \sum_{n=1}^N e^{-j\omega T_C J_n} \{1 - e^{-j\omega T_C x_n}\}. \quad (12)$$

The power spectral density of the inductor current is defined as $|A(\omega)|^2$.

When $\omega = m\omega_c$, where ω_c is clock angular frequency, one has

$$A_m = -\frac{\bar{V}_O}{\omega^2 L} \lim_{N \rightarrow \infty} \frac{1}{T_N} \sum_{n=1}^N 1 - e^{-2j\pi m x_n}, \quad (13)$$

where A_m stands for the peak values.

According to Birkhoff's ergodic theory [12], a mapping f , which is of invariant density, satisfies the following relationship,

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \phi(f^{n-1}(x)) = \int_0^\alpha \phi(y) \rho(y) dy. \quad (14)$$

Thus, $\tilde{A}_m \triangleq |A_m|^2$ can be expressed by the invariant density $\rho(x)$ as

$$\tilde{A}_m = |A_m|^2 = \left[-\frac{\bar{V}_O}{m^2 \omega_c^2 L \langle T \rangle} \right]^2 \times \left[\left(\int_0^\alpha \cos 2\pi m x dx - 1 \right)^2 + \left(\int_0^\alpha \sin 2\pi m x dx \right)^2 \right], \quad (15)$$

where

$$\langle T \rangle = \lim_{N \rightarrow \infty} \frac{T_N}{N} = T_C \left(1 + \lim_{N \rightarrow \infty} \frac{1}{N} \lfloor x_n \rfloor \right).$$

A comparison of the power spectral densities calculated by (13) without using the invariant density and by (15) by using the invariant density is illustrated in Fig. 9, and shows that both have almost the same accuracy, but the calculation by using the invariant density takes much shorter time because (13), which includes an exponent operation, need to be calculated N times, and it requires $N \rightarrow \infty$; and (15) just need one-time calculation since invariant density has been known.

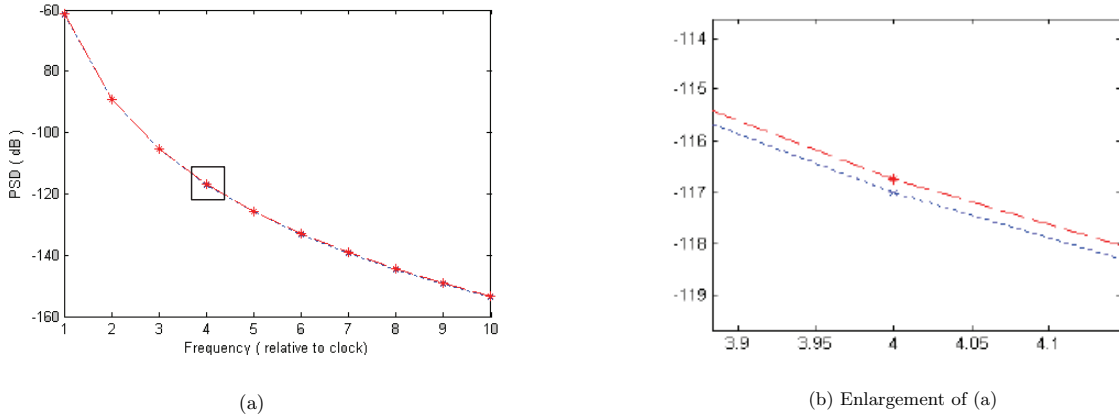


Fig. 9: Comparison of (13), shown as “+”, and (15), shown as “x”

6.2 Average switching frequency

Chaos control in DC-DC converter can not only reduce electromagnetic interference (EMI) of the DC-DC converter [13, 14, 15], but also reduce the average switching frequency, which is very important for reducing the switching loss and increasing the stability of DC-DC converter. The average switching frequency can be calculated with the invariant density.

For a Boost converter, as shown in Fig.1, if it operates in order, one can think that the total increment of the inductor current $\Delta_{i+(total)}$ is equal to the total decrement of the inductor current $\Delta_{i-(total)}$ for a relatively long time, namely, $\Delta_{i+(total)} = \Delta_{i-(total)}$ as shown in Fig. 10.

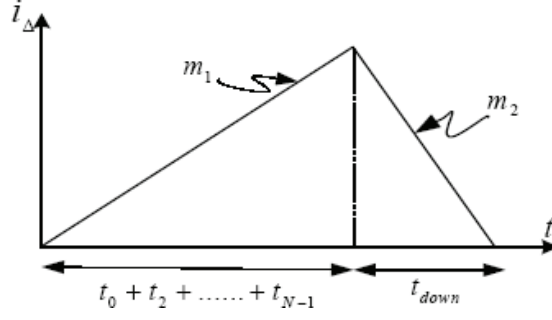


Fig. 10: equivalent figure

From Fig. 2, the total time corresponding to the increasing inductor current is $(t_0 + t_1 + \dots + t_{N-1})$, then the total time corresponding to the decreasing inductor current, t_{down} , can be obtained by,

$$(t_0 + t_1 + \dots + t_{N-1})m_1 = t_{down}m_2, \quad (16)$$

where $m_1 = \frac{V_I}{L}$ and $m_2 = \frac{\bar{V}_O - V_I}{L}$ are the rates of increment and decrement of the inductor current, respectively. Then, t_{down} can be obtained from (16) as

$$t_{down} = \frac{m_1}{m_2}(t_0 + t_1 + \dots + t_{N-1}) = \frac{1}{\alpha}(t_0 + t_1 + \dots + t_{N-1}), \quad (17)$$

and the total time of N -times switchings is

$$T_N = (1 + \frac{1}{\alpha})(t_0 + t_1 + \dots + t_{N-1}) = (1 + \frac{1}{\alpha})(x_0 + x_1 + \dots + x_{N-1})T_C. \quad (18)$$

Thus, the total number of clock cycle, denoted by L , is

$$L = \frac{T_N}{T_C} = (1 + \frac{1}{\alpha})(x_0 + x_1 + \dots + x_{N-1}), \quad (19)$$

and the total times of switching is N .

The average switching frequency is defined as [16],

$$\langle s \rangle = \lim_{N \rightarrow \infty} \frac{N}{L} = \lim_{N \rightarrow \infty} \frac{N}{(1 + \frac{1}{\alpha})(x_0 + x_1 + \dots + x_{N-1})} = \frac{1}{(1 + \frac{1}{\alpha})(\int_0^\alpha \rho(x) dx)}. \quad (20)$$

To simplify the analysis, choose α to be an integer larger than 1. By the chaotic mapping, it is easy to find that $\rho(x) = \frac{1}{\alpha}$ for the integer $\alpha \geq 1$. Then the average switching frequency can be got as

$$\langle s \rangle = \frac{2}{1 + \alpha}. \quad (21)$$

From (21), it is obvious that $\langle s \rangle = 1$ when $\alpha = 1$, implying that the Boost converter runs periodically; and $\langle s \rangle < 1$ when $\alpha > 1$. The Boost converter will operate in a chaotic mode when $\alpha > 1$, by which the Boost converter has a low average switching frequency and low switching loss. Further, as α increases, the average switching frequency decreases.

6.3 Parameter design for the Boost converter with invariant density

In designing a DC-DC converter, for instance, the Boost converter shown in Fig. 1, one need know the value of reference current I_{ref} . Generally speaking, the values of the input voltage and the output voltage are known conditions. According to [11], I_{ref} can be calculated from the following formula

$$\bar{V}_O^3 + \bar{V}_O(\frac{V_I T_c}{2L} - I_{ref})RV_I - \frac{RT_c V_I^3}{2L} = 0. \quad (22)$$

By the invariant density, one can accurately design the parameters for a chaotic DC-DC converter. To simplify the calculation, α samples integers from 2 to 10, because when $\alpha > 1$ takes integers, the corresponding invariant density is $\frac{1}{\alpha}$.

Denote the quantity of electric charge through the diode D at the n th time as $Q(x_n)$. Referring to the Fig. 2 and using the physical definition of quantity of electric charge, one has

$$Q(x_n) = (I_{ref} - \frac{m_2(1 + \lfloor x_n \rfloor - x_n)T_c}{2})(1 + \lfloor x_n \rfloor - x_n)T_c. \quad (23)$$

Using Birkhoff's ergodic theory and the invariant density, one can get

$$\langle T \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} T_c(1 + \lfloor x_n \rfloor) = T_c \int_0^\alpha (1 + \lfloor x \rfloor) \rho(x) dx = \frac{\alpha + 1}{2} T_c, \quad (24)$$

and

$$\langle Q \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} Q(x_n) = \int_0^\alpha Q(x) f(x) dx = \frac{1}{2} I_{ref} T_c - \frac{m_2 T_c^2}{6}. \quad (25)$$

Because of

$$\bar{I}_D = \frac{\bar{Q}}{\bar{T}}, \bar{Q} = \langle Q \rangle, \bar{T} = \langle T \rangle, \bar{I}_D = \frac{\bar{V}_O}{R}, \text{ and } \bar{V}_O = (1 + \alpha)V_I, \quad (26)$$

the reference current I_{ref} can be expressed as

$$I_{ref} = \frac{(1 + \alpha)^2 V_I}{R} + \frac{\alpha V_I T_c}{3L}. \quad (27)$$

A comparison of I_{ref} 's calculated by (22), by (27) using the invariant density, and by experiment, as shown in Fig. 11, shows that the estimation of I_{ref} with the invariant density is much more accurate.

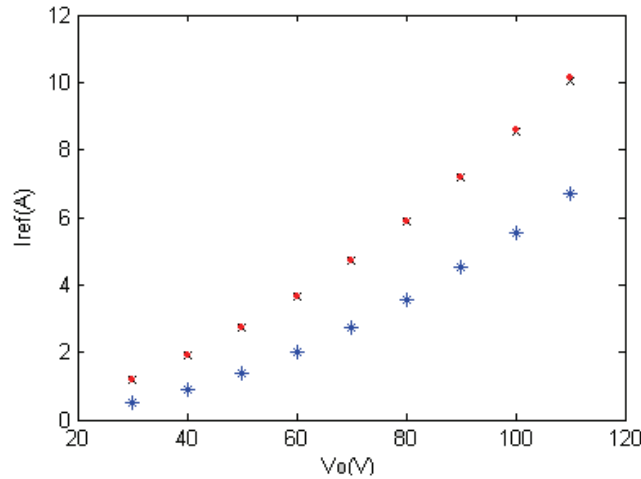


Fig. 11: Comparison of I_{ref} 's obtained by (22) ("*"), by (27) ("x"), and by experiment (".")

7 Conclusions

In this paper, a one-dimensional chaotic mapping of the DC-DC converter was derived, and the eigenvector method from probability theory was proposed to calculate the invariant density of the

chaotic mapping since chaos has the random-like characteristic. Further, the invariant density was used to calculate the PSD of the input of the DC-DC converter. Moreover, the calculation result of the average switching frequency with invariant density implies that when a DC-DC converter works in a chaotic mode, the average switching frequency is lower than that when it works in a periodic mode. Consequently, the switching loss of the DC-DC converter can be reduced when it works in a chaotic mode. Furthermore, the invariant density can be used to accurately design the parameters of DC-DC converters. Finally, the simulation results have illustrated the effectiveness of the eigenvector method.

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